

Calculations of the Effective Permittivity of a Periodic Array of Wires and the Left-Handed Materials.

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Abstract

We present finite-differences time- domain calculations of the effective permittivity of a periodic array of Cu strip wires that are an ingredient needed in metamaterials for obtaining left-handed materials. We use the Cu frequency dependent permittivity. The result is critically dependent of the wire thickness. In particular for the wires of $0.003cm$ thickness used in metamaterials experiments the imaginary part of the effective permittivity dominates over the real part, which is slightly positive at $11GHz$. Wires 10 - 20 times thicker may provided a good transparent left-handed material.

Studies of photonic band gaps have shown that for a square periodic array of wires that there is a band gap at certain wires thickness [1,3] when the frequency of the radiation $f < c/2a$; i.e a cut-off frequency, where c and a are the speed of light and the size of the periodicity, assuming that the medium where the wires are immersed is vacuum. This means that the effective permittivity of the medium $\epsilon(\omega)$, $\omega = 2\pi f$, is negative. Then periodic arrays of Cu wires are used in metamaterials to obtain negative permittivity, and split ring resonators are used to obtain negative permeability for left-handed materials (LHM) [4,5]. In fact in a recent paper[6] to determine the effective permittivity of metamaterials it has been reported: " A square array

of conducting wires, based on effective medium, is expected to exhibit the ideal frequency dependent plasmonic form of $\epsilon = 1 - \omega_{co}^2/\omega^2$, where ω_{co} is a cut-off frequency”, the cut off given above. Clearly this argument cannot be true in general, although we do deny that in certain cases, for relatively thick wires it may hold, and then the $Im(\epsilon)$ may be smaller than $Re(\epsilon)$. This is the case when the wire thickness is much much larger than the skin depth in the wires at the given frequency, where the calculations [1-3,6] have been done. However when the thickness is smaller than the skin depth the medium is obviously transparent and dielectric and the above expression for ϵ does not hold. Also when the thickness is of the order of the skin depth there should be a region where the expression above does not hold and in addition $Im(\epsilon)$ is the dominant and relevant part. There is no way to argue against this part of physics. In fact Walser, Valanju and Valanju has stated that for relative thin wires the losses, the imaginary part, dominates the permittivity of the wires [7] Recently we have argued [8] that in the experiments performed in metamaterials to obtain LHM, using wires $0.003cm$ thick we were in this last case where the imaginary part of the permittivity dominates over the real part and then there is not LHM because of losses, no observation of negative index of refraction can be claimed in a wedge like sample [9]. Nevertheless Markos et al [6] have insisted that thin $0.003cm$ or thick $0.1cm$ wires should have similar permittivities, denying our conclusions [8]. But interesting enough, all the arguments they used is that even if they can not calculate for thin wires they expected no changes with respect to thick wires [6,10]!!. So that nobody have calculating the behaviour of these wires as varying the thickness. Therefore it is crucial and very important in the problem of LHM to perform calculations of the effective permittivity of the medium for a periodic array of Cu wires by using the experimental $\epsilon_{Cu}(\omega)$ as the input ingredient for the calculation in order to clarify the above problem and simulate the best sizes for obtaining LHM.

We have performed, precisely, the above calculation using the finite-difference time-domain method [11] and the experimental values of the frequency dependent permittivity for Cu [12]. We show that for thin $0.003cm$ wires the imaginary part dominates and the real part is slightly positive for $11GHz$, so that no LHM material is obtained for this frequency. However for wires 10-20 time larger the real part of the permittivity is negative with a small imaginary part, and then one could obtain a good transparent LHM.

To perform the calculations we used as in the experiments [4,5] $a = 0.5cm$

and the thickness d of the wires was changed to see its effect on $\epsilon(\omega)$. The FDTD calculations Mur's first order absorbing conditions and s-polarized radiation, i.e the electric field parallel to the wires. The $\epsilon_{Cu}(\omega)$ is obtained by fitting the existing experimental data to obtain a plasmon frequency ω_p and the damping γ , and thus we have:

$$\epsilon_{Cu}(\omega) = 1 - \omega_p^2/(\omega^2 + i\gamma\omega) \quad (1)$$

The inset in Fig1 shows the plot of the data [11] and the best fitting for eq.(1) is obtained $f_p = 8.ev$ and $\gamma = 0.2$ respectively. The imaginary and real parts are given by the dashed and continuous lines respectively, adjusting very well to the data.

With the above permitivity for the Cu wires we have all the ingredients needed when the wires conform a periodic array of square prisms of size d in a periodic lattice of period $a = 0.5cm$. Calculation for reflectivity, transnmitivity and absorption (R, T and A ; $A = 1 - R - T$) respectively are presented in Fig.1b to 3b for the wire thickness $d = 0.1, 0.02$ and $0.003cm$, the later corresponding to the experimental ones [4,5] and using 3 rows of wires. It can be seen that while the value of R is practically unity for $d = 0.1cm$, this is reduced gradually for 0.02 and $0.003cm$ and the values of T grow but specially the absorption is clearly increasing as the thickness is reduced. This is because for 0.003 thickness the skin depth is $0.0005 - 0.001cm$, i.e. of the same order as the wire thickness and then the absorption and losses are dominant over the real part of the permitivity.

By using the numerical simulations of Fig.1b to 3b one can fit these values to a permitivity using an effective medium by adjusting its transmitivities and reflectivities given by formula (4) of ref. [8] for the transmitivity, and by similar formula for reflectivity. Least square fitting of effective permitivity $\epsilon(\omega)$ taking the permeability equal to unity to the numerical FDTD simulations. Results are presented in Fig. 1a to 3a and clearly shown the following: (i) for $0.1cm$ thick wires the real part is negative and much larger than the imaginary part of $\epsilon(\omega)$ for all frequencies and its real part is negative; (ii) for $0.02cm$ thick wires the real and the imaginary parts are practically equal and the real part is negative, so that losses play a significant role; and (iii) for $0.003cm$ wires the imaginary part clearly dominates the real part that is always positive except for the values $4GHz < f < 9GHz$, in fact for $f = 11GHz$ that is the value where it is observed and band pass filter

$Re(\epsilon) = 0.1$. Therefore for thin 0.003 wires, even for 0.02cm thickness the proposed existence of a cut-off plasmonic expression for the effective permittivity is not correct. We should also say that this has been concluded theoretically by Walser et al [7] that reach similar results simply because for thin wires the imaginary part, losses, are dominant. The inset of Fig.3 shows the behaviour of $Re(\epsilon)$ and $Im(\epsilon)$ for versus d at the relevant frequency of 11GHz. We also have calculated for 2 rows of wires and the results (not shown) are practically the same for the effective permittivity.

In conclusion we have shown that the values of the effective permittivity of a square array of wires are critically dependant of wire thickness and that the plasmonic behaviour assumption is in general wrong but it is valid for the relatively large thickness described here. This would suggest that by using wires of thickness 10 to 20 times larger than used in the experiments of metamaterials so far[4,5] it may be possible to obtain good transparent LHM.

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FIGIURE CAPTIONS

Fig.1. Inset: experimental values of $\epsilon_{Cu}(\omega)$ [11] (symbols) and fitted imaginary and real parts for $f_p = 8eV$ and $\gamma = 0.2eV$. (a) $Im(\epsilon)$ and $Re(\epsilon)$ (ϵ is the effective medium permittivity) continuous and dashed lines respectively ($d = 0.1cm$) Real part is negative and dominates the imaginary part that is practically equal to zero, very small losses. (b) FDTD calculations for reflectivity, transnmitivity and absorption (R , T and A , $A = 1 - R - T$). From these simulations and using the effective medium theory the values in (a) are obtained.

Fig.2 (a) Same as in Fig.1a for $d = 0.02cm$. Real and imaginary part are of the sam order, losses are important. Real part is negative. (b) Same as in Fig.1b.

Fig.3 (a) Same as in Fig.1a for $d = 0.003cm$. Imaginary part dominates over the real that is positive, except for the region $4 - 9GHz$, but is very small. Losses are important. This is the thickness used in metamaterials so far [4,5] and, therefore they are not LHM at the frequencies $9 - 20GHz$. (b) the same as in Fig.1b. Notice that now absorption A is very important. Inset; values of the real (dashed), left scale, and imaginary (continuous), right scale, parts of ϵ versus d at $11GHz$.